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РАЗДЕЛ 2. МАТЕМАТИЧЕСКИЕ МЕТОДЫ ФИНАНСОВОГО АНАЛИЗА

STOCHASTIC APPROACH TO INTEGRATED MODELING OF CREDIT AND INTEREST RATE RISK

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The paper presents a review of stochastic framework for term structure modeling and shows comparative advantages of commonly used techniques. The main application of the research is coherent modeling of credit and interest rate risk for Euro zone issuers.

The basics of reduced-form models of credit risk come to modeling of an unpredictable time of default. The main element in a wide area of research is *default intensity*. Due to duality of default probability and discount curves in terms of term structure properties it’s reasonable to apply interest rate models in credit case. Thus, many popular methods for forward rates modeling, such as bootstrapping, parametric and spline approaches, are considered possible for credit risk estimation. This paper reviews stochastic modeling framework and shows limitations in case of small number of parameters. The main problem arises during consecutive two-step modeling of discount factor and default probability. Introduced systematic error can lead to drastic changes in output result. The proposed method of increasing the reliability of results is usage of complex nonparametric dynamic methods and integrated model of interest and credit risk.

Analysis of Cox-Ingersoll-Ross framework

The model introduced by Cox, Ingersoll & Ross (1985) describes the behavior of interest rates as a movement driven by only one source of market risk. According to the model, instantaneous forward rate r_t follows the following stochastic equation (*CIR process*):

$$\mathrm{d}r_t = k(\theta - r_t)\mathrm{d}t + \nu\sqrt{r_t}\mathrm{d}Z_t$$

and the discount function $d(t,T)$ at moment t has a simple analytical expression:

$$d(t,T) = E_Q e^{-\int_0^T r_t(\tau)\mathrm{d}\tau} = A(t,T)e^{-B(t,T)r_0},$$

where $A(t,T), B(t,T)$ are known functions, dependent on parameters of the model.

The same approach can be applied to modeling hazard rate λ_t and probability of default $P(t,T)$.

Credit risk component can be estimated from market price data for liquid bonds and corresponding CDS contracts. Consecutive two-stage modeling postulates estimation of term structure. Credit term structure is then fitted to CDS quotes via the pricing equation (par spread concept):

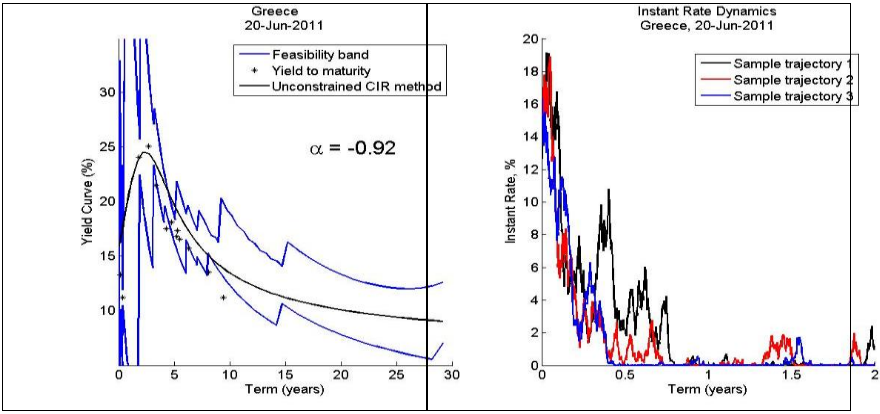
$$ParSpread_t = \frac{\int_0^T LGD d(t,\tau)\mathrm{d}(1-Q(\tau))}{\sum_{i=1}^{N} d(T_i)\delta(T_{i-1},T_i)Q(T_i) + \int_0^T d(\tau)\alpha(T_{I(\tau)},\tau)\mathrm{d}(1-Q(\tau))},$$

where $\delta(t_1,t_2)$ is a year fraction between the two dates, T_i are payment dates and $I(\tau)$ is the index of the last payment date before moment τ. For more information on this concept see, for example, Buhler & Trapp (2006).

The discussed methodic relies on the fact that the term structure of interest rates is estimated with high precision. This assumption needs verification for any given specification. Being a simple parametric method, CIR trajectories don't have enough degrees of freedom and, thus, in some cases can't fit given term structure. Besides, one set of CIR process parameters defines both the dynamics of instant rate and term structure curve. As a result we have to choose if we want to fit better dynamics or more accurate form for term structure. We'll illustrate the consequences of this for Euro zone bonds.

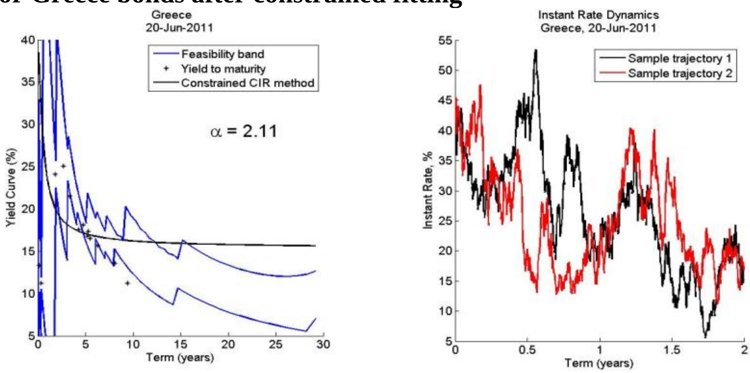
Fig. 1 illustrates the "good term structure" case. Jugged curves represent model-independent theoretical bounds for yield curve, described in Smirnov & Zakharov (2003). The dynamics provided by the same set of parameters k, θ, ν, r_0 has degenerate behavior and converges to zero due to the structure of the model and fitting procedure.

Fig.1. Yield curve (left) and provided instant rate dynamics (right) for Greece bonds



In Cox, Ingersoll & Ross (1985) constraint for parameters is obtained to guarantee non-degenerate dynamics of CIR trajectories: $\alpha = \frac{2k\theta}{\nu^2} - 1 > 0$

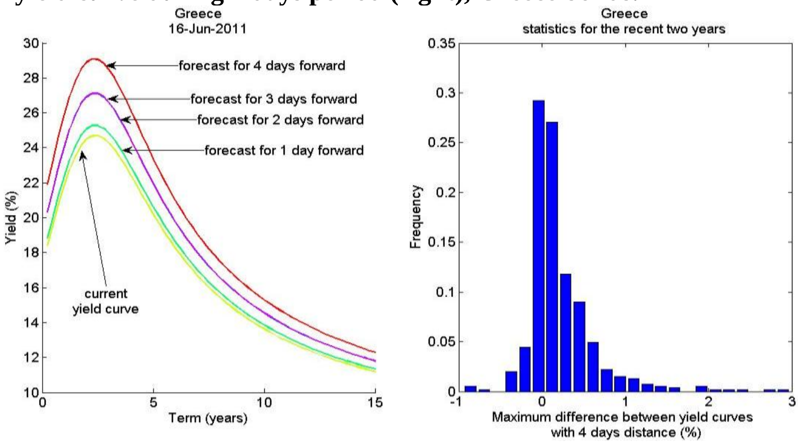
Fig.2. Yield curve (left) and provided instant rate dynamics (right) for Greece bonds after constrained fitting



Fitting results for Euro zone countries show that during the last 2 years degeneracy condition was satisfied in **less than 10%** of cases. It means that it cannot be ignored during fitting and considered satisfied as another assumption. Results for constrained CIR fitting are presented in Fig. 2. Originally optimal solution was archived for degenerate parameters ($\alpha = -0{,}92$) so the constrained optimum gives less accurate and sometimes even inappropriate results.

Another approach to estimating CIR parameters is fitting instant rate dynamics. As mentioned above, predefined form of corresponding yield curve can lead to unrealistic term structure estimates. The dynamics of term structure curves is not reliable due to stochastic nature of instant rate and high sensitivity to its value. Analysis of Euro zone data shows that even for a short horizon yield curve dynamics can be non-typical: yield "jumps" up to **5% in 4 days** while typical historical change for such period almost never exceeds **2%** as shown in Fig. 3.

Fig.3. Yield curve forecast (left) and histogram of maximum change in yield curve during 4 days period (right), Greece bonds.



Previous results show that systematic error is introduced during the first step of two-stage estimate of credit risk. The only possible solution of the problem (for two-stage algorithm) is using more complex and accurate techniques such as non-parametric methods.

Heath-Jarrow-Morton framework

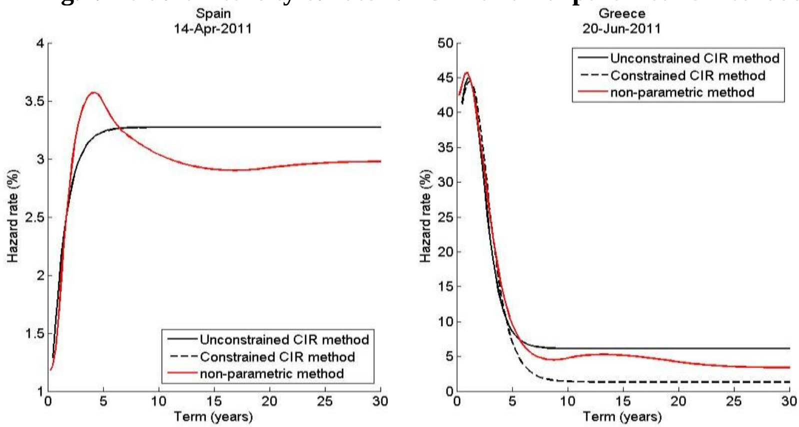
Another solution is the use of a doubly stochastic non-consecutive model like the one developed by Filipovic (2011). Our alternative is to specify another type of a doubly stochastic model when both spot forward rate $\boldsymbol{r_t}$ and hazard rate $\boldsymbol{\lambda_t}$ evolve according to an infinite-dimensional HJM-style equation:

$$\begin{cases} dr_t = (Dr_t + \alpha_t)dt + \sum_{j=1}^{\infty} \tilde{\sigma}_t^j d\beta_t^j \\ d\lambda_t = (D\lambda_t + \tilde{\alpha}_t)dt + \sum_{j=1}^{\infty} \tilde{\nu}_t^j d\tilde{\beta}_t^j \end{cases}$$

When these processes are assumed to be independent it reduces to separate non-parametric estimation of spot forward rates and hazard rate as the one described by Lapshin (2009). The exact dynamic parameters are unimportant since the estimation only involves a snapshot of the market. An infinite-dimensional model is essentially equivalent to a nonparametric estimate of spot forward rate and hazard rates. Yet, the underlying infinite-dimensional model is needed because it serves as a guarantee for the approach to be internally consistent and arbitrage-free for use on an everyday basis.

According to the model, we have conducted nonparametric estimates of interest and hazard rates. Fig. 4 compares the results of default intensity (or hazard rate) two-stage modeling using CIR methods and nonparametric techniques.

Fig.4. Default intensity curves for CIR and nonparametric methods



Obtained results show greater sensitivity to market data for nonparametric method and should provide reasonable results for forecasts due to the construction of the model. The following procedure can be proposed to estimate the actual accuracy of hazard rate structure:

1. Extraction of hazard rate term structure,

2. Bond model prices calculation and accuracy test using bid-ask quotes.

Conclusion

We have presented strong evidence of standard approach to credit and interest risk modeling being insufficient. Bootstrapping is well-known for producing awkward rates (e.g. negative) and CIR-like models tend to introduce significant model-inflicted errors while incapable of replicating complex shapes of forward interest rates and/or hazard rates.

We show that the use of more complex models may help to develop advanced fitting procedures for estimating term structure of interest rates and hazard rates which allows more adequate pricing and marking-to-market of risky bonds and derivative instruments.

Also these models may be used to better understand the role liquidity plays in pricing (risky) bonds by providing a nonparametric alternative technique of pricing an ideally liquid risky bond given CDS prices. Comparing model and real prices via bid-ask spreads may yield invaluable information on bond pricing mechanisms.

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