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The study of RQ-system with unreliable device

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Abstract

The paper studies a single-line RQ Queuing system M/M/1 with an incoming Poisson flow. Requests arrive at the system input, and if the device is free, then they are serviced. If the device is busy, then the requests go to the source of repeated calls and waiting for service. After a delay in the source of repeated calls, the request tries to occupy the device again, and if it is free, the incoming request takes it for service. It is assumed that the server is "unreliable", that is, it may fail, and then it is sent for repair. An analytical and numerical solution of this system is obtained by the method of generating functions.

Keywords: Queuing system, RQ-system, unreliable device;

1. Introduction

Mathematical models of RQ-systems are widely used in the analysis of processes in production and service systems, telephone systems, and computer complexes. A characteristic feature of these models is the ability to retransmit distorted messages when the device is busy, and an incoming request leaves the waiting area (orbit) and after some random intervals it repeats an attempt to get the service [1].

In this paper, a single-channel RQ queuing system with an unreliable device is studied. Queuing systems are called unreliable if their devices may fail over time and require repair work, and only after which they can restore query serving as a new system.

Today a large number of scientific papers are devoted to the study of queuing systems with unreliable servicing devices, a review of which is given in the paper. "A brief review of work in the field of research of queuing systems with unreliable servicing devices", written by Dudin A.N., Dudin S.A [2]. At the same time, there is a disadvantage in most papers. These are very strict assumptions about the exponential distribution of time between the requests and breakdowns, the time of query serving, the duration of the repair work [3].

2. Description of the model and problem statement

I want first to study a single line RQ system with an unreliable instrument (Figure 1). The system input receives the simplest flow of requests with the intensity λ. The service time of each request is exponentially distributed with the parameter μ_1. If the received request finds the device free, it takes it for maintenance; otherwise the request goes to the source for repeated calls or to the orbit, where it waits for some random time with the parameter σ, distributed according to the exponential law. From the source of repeated calls, after a random delay, the request returns to the service device with a second attempt to capture it. If the device is free, the request occupies it for maintenance, otherwise the request is instantly returned to the source of repeated calls to implement the next delay.

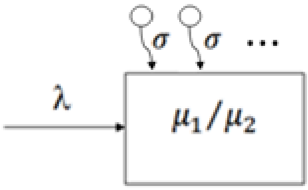

Figure 1. Model of system

It is assumed that the server is unreliable, that is, the uptime is a random variable exponentially distributed with the parameter γ_1, if the server is out of action; and with the parameter γ_2, if it is busy with maintenance. When the server fails, it is immediately sent for repairing and the recovery time is exponentially distributed with the parameter μ_2.

When the server is in operative, all incoming requests immediately go to the orbit.

Let $i(t)$ be the number of requests in orbit at time t, and $k(t)$ determines the state of the device:

$$k(t)=\begin{cases}0, \text{if the device is free,}\\ 1, \text{if the device is busy,}\\ 2, \text{if the device is under repair.}\end{cases}$$

The process of changing the states of a given system in time $\{k(t), i(t)\}$ is a two-dimensional Markov chain.

It is required to find the stationary probability distribution of the number of requests in the source of repeated calls taking into account the state of the device

$$P_k(i,t)=P\{k(t)=k,\ i(t)=i\};\ \mathrm{k}=0,1,2;\ i=0,1,2\ldots$$

3. Kolmogorov system of differential equations

For the probability distribution $P_k(i,t)$, we compose a system of Kolmogorov differential equations

$$\begin{cases}\partial P_0(i,t)/\partial t=-(\lambda+i\sigma+\gamma_1)P_0(i,t)+\mu_1P_1(i,t)+\mu_2P_2(i,t),\\ \partial P_1(i,t)/\partial t=-(\lambda+\mu_1+\gamma_2)P_1(i,t)+\lambda P_0(i,t)+(i+1)\sigma P_0(i+1,t)+\lambda P_1(i-1,t),\\ \partial P_2(i,t)/\partial t=-(\lambda+\mu_2)P_2(i,t)+\gamma_1P_0(i,t)+\gamma_2P_1(i-1,t)+\lambda P_2(i-1,t).\end{cases} \qquad (1)$$

Let us denote $P_k(i)=\lim\limits_{t\to\infty}P_k(i,t)$. Then, in stationary mode, the system (1) will take the form

$$
\begin{cases}
-\left(\lambda+i\sigma+\gamma_1\right)P_0\left(i\right)+\mu_1P_1\left(i\right)+\mu_2P_2\left(i\right)=0,\\
-\left(\lambda+\mu_1+\gamma_2\right)P_1\left(i\right)+\lambda P_0\left(i\right)+\left(i+1\right)\sigma P_0\left(i+1\right)+\lambda P_1\left(i-1\right)=0,\\
-\left(\lambda+\mu_2\right)P_2\left(i\right)+\gamma_1P_0\left(i\right)+\gamma_2P_1\left(i-1\right)+\lambda P_2\left(i-1\right)=0.
\end{cases}
\tag{2}
$$

4. System solution by the method of generating functions

We introduce generating functions $F_k\left(z\right)=\sum_{i=0}^{\infty}z^iP_k\left(i\right)$, then from (2) we obtain

$$
\begin{cases}
-\left(\lambda+\gamma_1\right)F_0\left(z\right)-\sigma z\partial F_0\left(z\right)/\partial z+\mu_1F_1\left(z\right)+\mu_2F_2\left(z\right)=0,\\
-\left(\lambda+\mu_1+\gamma_2\right)F_1\left(z\right)+F_0\left(z\right)+\sigma\partial F_0\left(z\right)/\partial z+\lambda zF_1\left(z\right)=0,\\
-\left(\lambda+\mu_2\right)F_2\left(z\right)+\gamma_1F_0\left(z\right)+\gamma_2zF_1\left(z\right)+\lambda zF_2\left(z\right)=0.
\end{cases}
\tag{3}
$$

The solution of system (3), taking into account the normalization condition $F_1\left(1\right)+F_2\left(1\right)+F_0\left(1\right)=1$has the form $F\left(z\right)=F_0\left(z\right)+F_1\left(z\right)+F_2\left(z\right)$, where

$$
F_0\left(z\right)=R_0\exp\left\{-\frac{\lambda}{\sigma}\int_z^1\left(\frac{\left(\lambda+\mu_1+\gamma_2-\lambda z\right)\left(\lambda+\mu_2-\lambda z+\gamma_1\right)}{\left(\mu_1-\lambda z\right)\left(\lambda+\mu_2\right)-\lambda z\left(\gamma_2+\mu_1-\lambda z\right)}-1\right)dz\right\}
\tag{4}
$$

$$
R_0=F_0\left(1\right)=\frac{\mu_1\mu_2-\mu_2\lambda-\gamma_2\lambda}{\mu_1\left(\mu_2+\gamma_1\right)},\quad F_1\left(z\right)=F_0\left(z\right)\frac{\lambda^2+\mu_2\lambda-\lambda^2z+\gamma_1\lambda}{\left(\mu_1-\lambda z\right)\left(\lambda+\mu_2\right)-\lambda z\left(\gamma_2+\mu_1-\lambda z\right)}
\tag{5}
$$

$$
F_2\left(z\right)=F_0\left(z\right)\frac{\gamma_2z\lambda+\gamma_1\mu_1-\gamma_1\lambda z}{\left(\mu_1-\lambda z\right)\left(\lambda+\mu_2\right)-\lambda z\left(\gamma_2+\mu_1-\lambda z\right)}
\tag{6}
$$

5. Probability distribution of the number of requests

In order to find the probability distribution of the number of requests in the orbit, it is necessary to differentiate the generating function found by formulas (4), (5), (6).

$$
P\left(i\right)=\frac{1}{i!}\frac{\partial^iF\left(z\right)}{\partial z^i}
$$

Let us consider the system with the following parameters: $\mu_1=7,\ \mu_2=1,\ \gamma_1=0.03,\ \gamma_2=0.03,\ \lambda=3,\ \sigma=1.$

The probability distribution of the number of requests in orbit is shown in Figure 2.

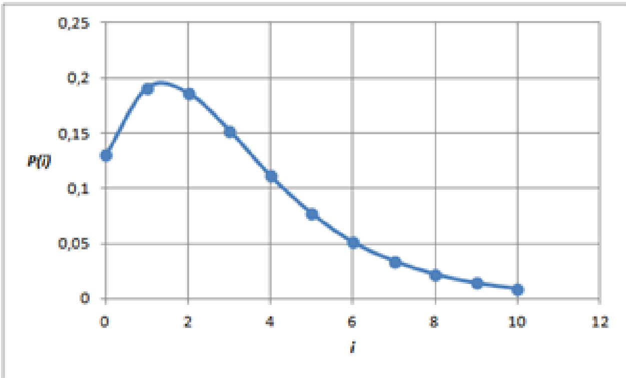

Figure 2. Probability distribution of the number of requests

Let us find the main characteristics of the system: mathematical expectation and variance according to formulas.

$$M\{i(t)\} = F'(1) = 3.074$$

$$D\{i(t)\} = F''(1) + F'(1) - (F'(1))^2 = 7.586$$

6. Conclusion

In our work, we investigated the RQ-system with an unreliable device. The system of Kolmogorov equations is solved by the generating function method. The stationary distribution of the probabilities of the number of request in the orbit and the main characteristics of the system are obtained.

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